# Problems, Tutorial 4 

## Space physics EF2240, 2014

1. 

a) Figure 1a shows Voyager 2 measurements of the electron density of the Jupiter dayside ionosphere as a function of altitude above the surface. The surface of Jupiter is defined as the region where the pressure of the neutral gas is $1 \mathrm{bar}=100 \mathrm{kPa}$. It is still not known if the main ionospheric layer with the peak at around 1900 km altitude is associated with ionization of atomic hydrogen or molecular hydrogen. Supported by a calculation, state which alternative you believe is correct.

In Table 1 b are given values of ionization cross sections $\bar{\sigma}$ (the same as the absorption coefficient called $a_{a}$ in Fälthammar), for various wavelength intervals of the incoming solar EUV radiation. Use the values in the range 504-804 $\AA$. Assume that the layer can be described by a Chapman profile, and that the neutral atmosphere is exclusively made up of either molecular or atomic hydrogen. Assume (somewhat unrealistically) that the temperature in the Jupiter atmosphere is constant with altitude and has a value of 300 K . The gravitational acceleration at the Jupiter surface is 23.1 $\mathrm{ms}^{-2}$.


Electron number density in Jupiter's ionosphere derived from Voyager 2 radio measurements at occultation exit. Altitude reference is the 1-bar equipotential surface defined by Lindal et al. [1981]. (Hinson et al., 1998)

Mean Cross Sections and Fluxes

| $\bar{\sigma}\left(\mathrm{H}_{2}\right)(\lambda<504)$ | $1.7 \times 10^{-18} \mathrm{~cm}^{2}$ |
| :--- | :--- |
| $\bar{\sigma}\left(\mathrm{H}_{2}\right)(804>\lambda>504)$ | $6.1 \times 10^{-18} \mathrm{~cm}^{2}$ |
| $\bar{\sigma}(\mathrm{He})(\lambda<504)$ | $3.3 \times 10^{-18} \mathrm{~cm}^{2}$ |
| $\bar{\sigma}(\mathrm{H})(912>\lambda>804)$ | $5.0 \times 10^{-18} \mathrm{~cm}^{2}$ |
| $\bar{\sigma}(\mathrm{H})(804>\lambda>504)$ | $3.0 \times 10^{-18} \mathrm{~cm}^{2}$ |
| $\bar{\sigma}(\mathrm{H})(\lambda<504)$ | $6.0 \times 10^{-19} \mathrm{~cm}^{2}$ |
| $\overline{\mathcal{F}}_{\infty}$ | $(\lambda<504)$ |
| $\overline{\mathcal{F}}_{\infty}$ | $(804>\lambda>504)$ |

Table 1b (Gross and Rasool, 1964)
b) Assuming that the main peak electron density maximum is representative for the whole dayside, estimate the minimum nightside electron density at 1900 km altitude, assuming that the ionosphere is corotating with Jupiter. Calculate the result for both assumptions; the layer consisting of either atomic and molecular hydrogen. The recombination coefficients for these two cases are given in Table 2c (entries 9 and 13), in units of $\mathrm{cm}^{3} \mathrm{~s}^{-1}$. What can you say about the nightside electron density compared to the dayside one?

| Recombination | (9) $\mathrm{H}_{2}+\mathrm{e} \rightarrow \mathrm{H}^{\prime}+\mathrm{H}^{\prime \prime}$ | $\alpha_{9}=10^{-8}$ |
| :--- | :--- | :---: |
|  | (10) $\mathrm{HeH}^{+}+\mathrm{e} \rightarrow \mathrm{He}+\mathrm{H}$ | $\alpha_{10}=10^{-8}$ |
| (11) $\mathrm{He}^{+}+\mathrm{e} \rightarrow \mathrm{He}$ | $\alpha_{11}=10^{-12}$ |  |
|  |  |  |
|  | (12) $\mathrm{H}_{3}++\mathrm{e} \rightarrow \mathrm{H}_{2}+\mathrm{H}$ | $\alpha_{12}=10^{-8}$ |
|  | (13) $\mathrm{H}^{+}+\mathrm{e} \rightarrow \mathrm{H}$ | $\alpha_{13}=10^{-12}$ |
| (14) $\mathrm{H}+\mathrm{H}+\mathrm{X} \rightarrow \mathrm{H}_{2}+\mathrm{X}$ | $\alpha_{14}=10^{-32}$ |  |
|  | (15) $\mathrm{H}+\mathrm{e} \rightarrow \mathrm{H}^{-}+h \nu$ | $\alpha_{15}=10^{-16}$ |

Table 2c (Gross and Rasool, 1964)
(From Exam, Oct., 2009.)
2. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$
\mathbf{B}=\left\{\begin{array}{lll}
-B_{0} \hat{\mathbf{x}} & , & y<-a \\
B_{0} \hat{\mathbf{x}} \frac{3 a^{2} y-y^{3}}{2 a^{3}} & , & -a \leq y \leq a \\
B_{0} \hat{\mathbf{x}} & , & y>a
\end{array}\right.
$$

where $B_{0}=10 \mathrm{nT}, \mathrm{a}=2000 \mathrm{~km}$ and the coordinates are defined as in Figure 1.
a) Calculate the current density $j(x, y)$ (magnitudes and directions as functions of position). Also calculate the numerical value at $y=0$.
b) Calculate the total current floating in an $80 \mathrm{R}_{\mathrm{E}}$ long magnetotail.


Figure 2
3. Let the geomagnetic field be approximated by a dipole field. Assume that the magnetospheric plasma in the equatorial plane at $5 \mathrm{R}_{\mathrm{E}}$ is made up of protons with an energy of 1 eV and electrons with energy 10 keV . Both particle populations have a pitch angle of close to $90^{\circ}$, which means that they will not move far from the equatorial plane before they mirror. Calculate the grad- $B$ drift for both particle populations, the direction of the drift, and the time it takes them to drift around Earth once.
4. Estimate the voltage drop over the magnetosphere (in the east-west) direction, when the solar wind velocity is $350 \mathrm{~km} / \mathrm{s}$, and $\mathrm{IMF} \mathrm{Bz}=-7 \mathrm{nT}$.
5. En electron is situated right above the magnetic pole at an altitude of 10000 km and is moving down towards Earth with a pitch angle of $15^{\circ}$. At what altitude will the electron mirror and start moving upwards?

